Int. J. Heat Mass Transfer. Vol. 36, No. 18, pp. 4477–4485, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00

Wall functions for numerical simulation of turbulent natural convection along vertical plates

X. YUAN, A. MOSER and P. SUTER
Energy Systems Laboratory, Swiss Federal Institute of Technology Zurich, ETH Zentrum, CH-8092, Zurich, Switzerland

(*Received* 12 *April* 1993)

Abstract—This paper presents new wall functions of velocity and temperature for natural convection along vertical plates based on dimensional analysis and experimental data. Because only fluid properties and local parameters are included, the proposed wall functions are suitable for numerical simulation and are expected to be also valid in non-isothermal flows in cavities. Moreover, a certain analogy of length, velocity, and temperature scales between natural and forced convection has been found.

1. INTRODUCTION

NATURAL convection and buoyancy induced flows abound in nature and in our living environment. In the past 20 years computational fluid dynamics (CFD) in natural convection has made considerable progress. However, numerical methods are still not well enough established to model it. While the wall functions, the distributions near the wall, have been widely used in forced convection, proper wall functions for natural convection have still not been found. A very fine grid has to be used in the near-wall region in natural convection. It was recommended that at least 10 grid lines [1], or even 20 to 30 [2] are required in the near-wall region, which significantly increases the computing cost, especially in 3-D cases. It is significant to find new wall functions which are suitable for natural convection.

Contributions have been made to the measurement and analysis of turbulent natural convection along heated vertical surfaces. Cheesewright [3] reported his experimental data in the form of $\theta = f(\eta)$, and $u/(g\beta(T_w - T_0)x)^{0.5} = f(\eta)$, with $\eta = y\,Gr_x^{0.1}/x$. He mentioned that, taking $\eta = y\,Gr_x^{0.4}/x$ would give much better correlation over the inner part of the boundary layer but would give the wrong behaviour near the outer layer edge. Fujii *et al.* [4] proposed to use $\zeta = y\,Nu_x/x$ as the length parameter, which was employed later by Miyamoto and Okayama [5] for correlation of the temperature and velocity profiles of the whole boundary layer, and by Tsuji and Nagano [6] and Henkes [7] for correlation of temperature in the inner layer. George and Capp [8] divided the boundary layer into two sublayers—inner and outer sublayers, of which the inner one is a constant heat flux sublayer and by means of similarity analysis, they obtained the 1/3-power-law wall functions for the buoyant sublayer.

Cheesewright and Mirzai [9] emphasized that, the correlation of temperature data is insensitive to the wall shear stress but the velocity data is correlated by splitting it into a part dependent on the shear stress and a part directly dependent on the temperature field. Based on experiments, Tsuji and Nagano [6] derived their own wall functions, in which the maximum velocity was taken as velocity scale and the boundary thickness as length scale for the outer layer, and they mentioned the difficulties in finding the proper similarity variable for the profiles of mean velocity, mean temperature, and intensities of velocity and temperature fluctuations in the outer layer.

Henkes [7], to complement experimental data, calculated natural convection boundary layers up to $Ra = 10^{25}$ with k–ε turbulence models. From the calculated results, he obtained the wall functions, in which he used the same scales in the outer layer as Tsuji and Nagano.

Moser [10] reviewed the methods and problems in numerical simulation of air flows in buildings, and emphasized the need for proper wall functions for natural and mixed convection.

In this paper, a new correlation, based on dimensional analysis [11] and the experimental data, is discussed. And new wall functions for turbulent natural convection along vertical plates are presented.

2. WALL FUNCTIONS

The conservation equations for momentum and energy in boundary layers along vertical smooth plates can be expressed by:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y} - \overline{u'v'}\right) + g\beta(T - T_0) \quad (1)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y} - \overline{v'T'}\right). \quad (2)$$

NOMENCLATURE

a, b, c, d dimensionless constant
C_p specific heat at constant pressure [J kg^{-1} K^{-1}]
g gravitational acceleration [m s^{-2}]
Gr_x x-Grashof number, $x^3 g\beta(T_w - T_0)/\nu^2$
Nu_x x-Nusselt number, $xq_w/(\alpha\rho C_p(T_w - T_0))$
Pr Prandtl number, ν/α
R dimensionless parameter, u_q/u_τ
q_w wall heat flux [W m^{-2}]
T local mean temperature [K]
T_q heat flux temperature, $(q_w^3/g\beta\alpha(\rho C_p)^3)^{1/4}$ [K]
T_w temperature of the surface [K]
T_0 temperature of the outside of boundary layer [K]
T^+ dimensionless temperature, $(T_w - T)\rho C_p u_\tau/q_w$
T^* dimensionless temperature, $(T_w - T)/T_q$
T^{**} dimensionless temperature, $T^* R^a$
u mean velocity component in the streamwise direction [m s^{-1}]
u_q a velocity scale based on heat flux, $(g\beta\alpha q_w/(\rho C_p))^{1/4}$ [m s^{-1}]
u_τ friction velocity, $(\tau_w/\rho)^{1/2}$ [m s^{-1}]
u^+ dimensionless streamwise velocity, u/u_τ
u^* dimensionless streamwise velocity, u/u_q
u^{**} dimensionless streamwise velocity, $u^* R^c$
x distance in streamwise direction [m]
y distance normal to the wall [m]
y^+ local Reynolds number, yu_τ/ν
y^* dimensionless distance normal to the wall, yu_q/α
y^{**} dimensionless distance normal to the wall, $y^* R^d$
y_i^{**} dimensionless distance for the inner sublayer, $y^* R^2$
y_0^{**} dimensionless distance for the outer sublayer, $y^* R^6$.

Greek symbols
α thermal diffusivity [m^2 s^{-1}]
β gas expansion coefficient [K^{-1}]
θ dimensionless temperature, $(T - T_0)/(T_w - T_0)$
λ heat conductivity [W m^{-1} K^{-1}]
μ molecular (or dynamic) viscosity [N s m^{-2}]
ν kinematic viscosity [m^2 s^{-1}]
ρ density of fluid [kg m^{-3}]
τ_w wall shear stress [Pa].

Velocity, u, v, and temperature, T, depend on the position, (x, y), the boundary condition, $(T_w - T_0)$, and the fluid properties ($g\beta$, α, and ν), i.e.,

$$u = f(x, y, T_w - T_0, g\beta, \alpha, \nu) \tag{3}$$

$$T_w - T = f(x, y, T_w - T_0, g\beta, \alpha, \nu). \tag{4}$$

Other dependent variables are the wall heat flux, q_w, and the wall shear stress, τ_w. All experimental data about those may be cast into:

$$Nu_x \equiv \frac{q_w x}{\rho C_p \alpha (T_w - T_0)} = f(Gr_x, Pr) \tag{5}$$

$$\frac{\tau_w}{\rho u_s^2} = f(Gr_x, Pr) \tag{6}$$

where u_s is the velocity scale, for which as proposed by Tsuji and Nagano [6, 12] $u_s = [g\beta(T_w - T_0)\nu]^{1/3}$ and by Cheesewright and Mirzai [9] $u_s = [g\beta(T_w - T_0)x]^{1/2}$.

The profiles (3) and (4) are not in a form suitable for wall functions in a numerical finite-volume method because the streamwise distance, x, still appears. The idea now is to use the empirical information in expressions (5) and (6) to eliminate x and $(T_w - T_0)$ in (3) and (4). Equations (5) and (6) can be rewritten as the dimensional equations:

$$\frac{q_w}{\rho C_p} = f(x, T_w - T_0, g\beta, \alpha, \nu) \tag{7}$$

$$\frac{\tau_w}{\rho} = f(x, T_w - T_0, g\beta, \alpha, \nu) \tag{8}$$

which are equivalent to:

$$x = f\left(\frac{\tau_w}{\rho}, \frac{q_w}{\rho C_p}, g\beta, \alpha, \nu\right) \tag{9}$$

$$T_w - T_0 = f\left(\frac{\tau_w}{\rho}, \frac{q_w}{\rho C_p}, g\beta, \alpha, \nu\right). \tag{10}$$

Substituting equations (9) and (10) into equations (3) and (4), we have:

$$u = f\left(y, \frac{\tau_w}{\rho}, \frac{q_w}{\rho C_p}, g\beta, \alpha, \nu\right) \tag{11}$$

$$T_w - T = f\left(y, \frac{\tau_w}{\rho}, \frac{q_w}{\rho C_p}, g\beta, \alpha, \nu\right) \tag{12}$$

which can be considered as general velocity and temperature profiles in boundary layers along vertical plates. Next, we will discuss how to deduce the wall functions for velocity and temperature by means of dimensional analysis.

2.1. *Temperature wall function*

From the seven variables in equation (12), only four independent dimensionless parameters can be formed, we choose:

$$Pr \equiv \frac{\nu}{\alpha} \tag{13}$$

$$T^* \equiv \frac{T_w - T}{T_q} \tag{14}$$

$$y^* \equiv \frac{y u_q}{\alpha} \tag{15}$$

$$R \equiv \frac{u_q}{u_\tau} \tag{16}$$

where:

$$T_q \equiv \left(\frac{q_w^3}{g\beta\alpha(\rho C_p)^3}\right)^{1/4},$$

which we call *heat flux temperature*;

$$u_q \equiv \left(\frac{g\beta\alpha q_w}{\rho C_p}\right)^{1/4}; \quad \text{and} \quad u_\tau \equiv \left(\frac{\tau_w}{\rho}\right)^{1/2},$$

which is known as 'friction velocity'.

Equation (12) can be therefore written as:

$$f(T^*, y^*, R, Pr) = 0. \tag{17}$$

The dimensionless equation (17) should represent the wall function for temperature in non-dimensional terms, and hence, the temperature profile depends on two parameters, R and Pr. The actual shape of the function can be determined by comparison with measured profiles in three steps:

(1) For the case restricted to air only, the Prandtl number is really a constant, $Pr = 0.71$.

(2) The experimentally observed dependence on R is accounted for by suitable transformation of the y- and T-coordinates using powers of R as equations (19) and (20).

(3) The profile in transformed coordinates is finally curve-fitted to measured data.

We expect the following equation to correlate temperature profiles well:

$$T^{**} = f(y^{**}, Pr) \tag{18}$$

where:

$$T^{**} = T^* R^a \tag{19}$$

$$y^{**} = y^* R^b. \tag{20}$$

If $a = b = -1$, then $T^{**} = T^+, y^{**} = Pr\, y^+$, equation (18) becomes:

$$T^+ = f(y^+, Pr) \tag{21}$$

which is the temperature wall function for forced convection.†

For natural convection, we have to determine the values of a and b based on experimental data. Tsuji and Nagano [12] systematically presented their experimental data of turbulent natural convection of air along a vertical plate with constant wall temperature in the range of $1.55 \times 10^{10} \leqslant Gr_x \leqslant 1.80 \times 10^{11}$. It can be seen from Fig. 1 that $T^* = f(y^*)$, or equation (18) with $a = b = 0$, correlate the experimental data well. By means of curve-fitting, as shown in Fig. 2, we obtain temperature wall function as follows:

$$T^* = y^* \quad y^* \leqslant 1 \tag{22}$$

$$T^* = 1 + 1.36 \ln y^* - 0.135 \ln^2 y^* \quad 1 < y^* \leqslant 100 \tag{23}$$

$$T^* = 4.4 \quad y^* > 100. \tag{24}$$

The available experimental data of Cheesewright and Mirzai [9] ($Gr_x = 3.94 \times 10^{10}$, 2.05×10^{10}), and Cheesewright [3] ($Gr_x = 5.72 \times 10^{10}$) for air flows along vertical plates with constant wall temperature, and Miyamoto *et al.* [13] ($Gr_x = 2.37 \times 10^{10}$, 1.26×10^{11}, 1.90×10^{11}) for air flows along vertical plates with constant wall heat flux are also shown in Fig. 2. The function agrees with the experimental data well and is valid in the situations of both constant wall temperature and constant wall heat flux.

According to the definitions of T^* and y^*, equation (22) can be rewritten as:

$$q_w = \frac{\lambda(T_w - T)}{y} \quad y^* \leqslant 1 \tag{25}$$

which is identical to the expression of the temperature profile in the laminar sublayer and agrees with the comment of Tsuji and Nagano [6].

At the outer edge of the boundary layer the temperature becomes $T = T_0$, and the dimensionless temperature approaches T_0^*, i.e.

$$T_0^* = \frac{T_w - T_0}{T_q} = \left(\frac{Pr^2\, Gr_x}{Nu_x^3}\right)^{1/4}. \tag{26}$$

According to the following relationship proposed by Tsuji and Nagano [12]:

$$Nu_x = 0.11\, Gr_x^{1/3} \tag{27}$$

we have:

$$T_0^* = \left(\frac{Pr^2}{0.11^3}\right)^{1/4} = 4.4 \quad \text{for } Pr = 0.71 \tag{28}$$

which is consistent with equation (24).

The agreement of the correlation of temperature profiles with measurement both in forced and natural convection with appropriate values of the exponents a and b confirms that equation (18) is a proper formula for the temperature wall function.

2.2. *Velocity wall function*

Similarly, we can determine four independent dimensionless parameters from equation (11). Then equation (11) can be changed into dimensionless form, i.e.,

† It is possible that equation (18) contains the wall function for forced convection because the temperature in forced convection can also be expressed by equation (12).

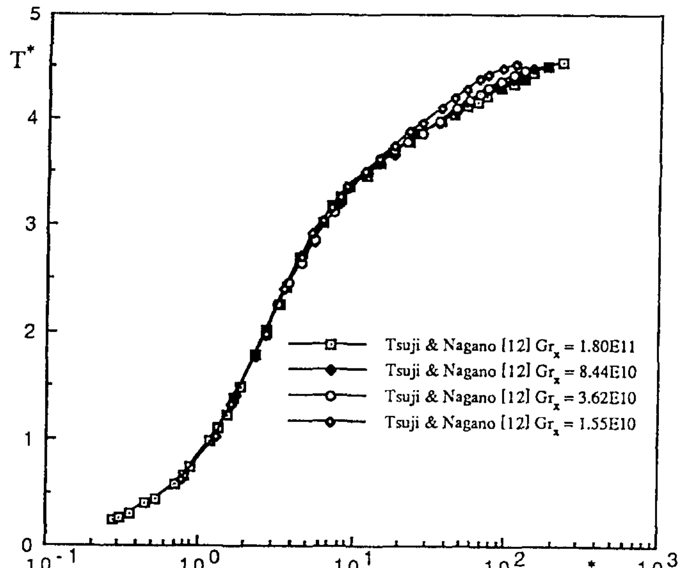


FIG. 1. The correlation of the temperature profile in the turbulent natural convection boundary layer along vertical plates.

$$f(u^*, y^*, R, Pr) = 0 \tag{29}$$

where

$$u^* \equiv \frac{u}{u_q}. \tag{30}$$

The following equation is expected to correlate velocity profile well:

$$u^{**} = f(y^{**}, Pr) \tag{31}$$

where

$$u^{**} = u^* R^c \tag{32}$$

$$y^{**} = y^* R^d. \tag{33}$$

When $c = 1$, and $d = -1$, then $u^{**} = u^+$, $y^{**} = Pr\, y^+$, and equation (31) becomes:

$$u^+ = f(y^+, Pr) \tag{34}$$

in which the velocity wall function for forced convection is contained.

For natural convection of air, c and d can be deter-

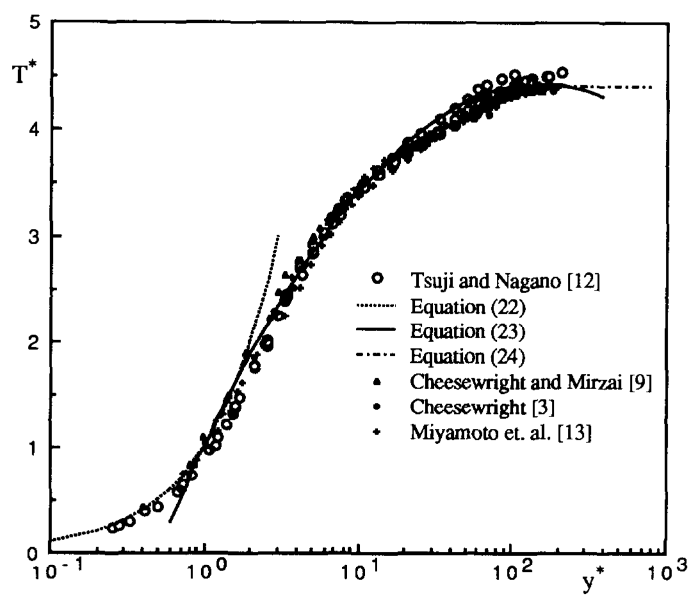


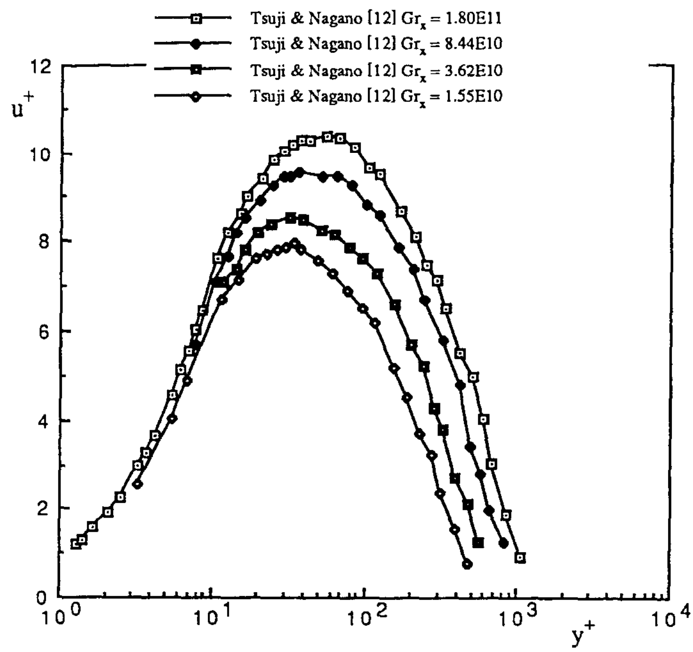
FIG. 2. The temperature wall function of turbulent natural convection along vertical plates.

mined based on the experimental data of Tsuji and Nagano [12]. Figure 3(a) shows that $u^+ = f(y^+)$ cannot correlate the experimental data. When we choose $c = 4$, the maximum of u^{**} is independent of Gr_x, as shown in Fig. 3(b), which is the essential condition to correlate the velocity profile. Then we get a dimensionless velocity parameter:

$$u^{**} = \frac{u_q^3 u}{u_\tau^4}. \tag{35}$$

Now we have to determine the value of d, but whatever the d value chosen, $u^{**} = f(y^{**})$ can still not correlate the experimental data well in the whole boundary layer. We have to divide the boundary layer into inner and outer sublayers as George and Capp [8], Tsuji and Nagano [6], and Henkes [7] did. The inner one is the region from the wall to the maximum velocity position, and the outer one is the rest region of the boundary layer. Figure 4 shows that $u^{**} = f(y^{**})$ correlates the experimental data well when $d = 2$ for

(a)



(b)

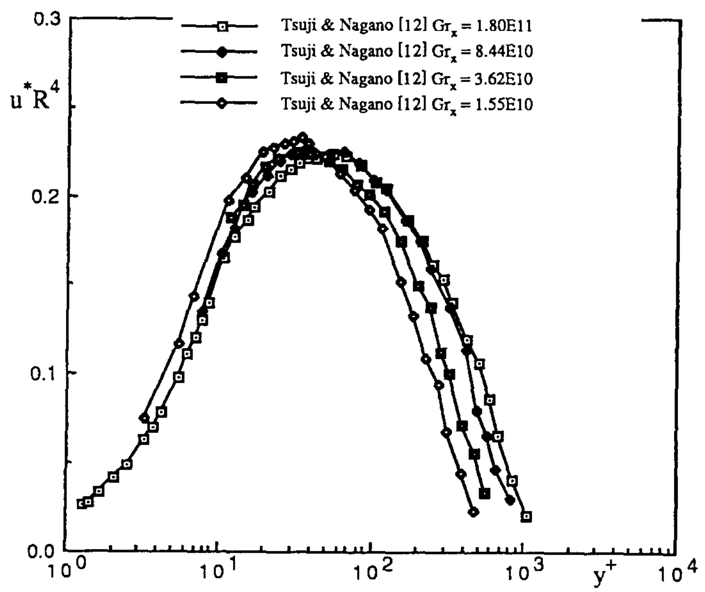
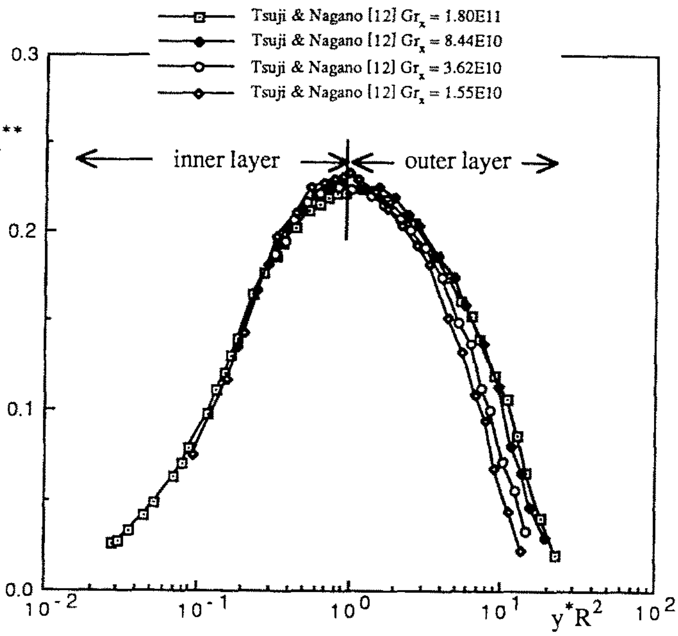


FIG. 3. The experimental data of turbulent natural convection boundary layer along vertical plates. (a) The experimental data in the form of $u^+ = f(y^+)$. (b) The experimental data in the form of $u^{**} = f(y^+)$.

(a)



(b)

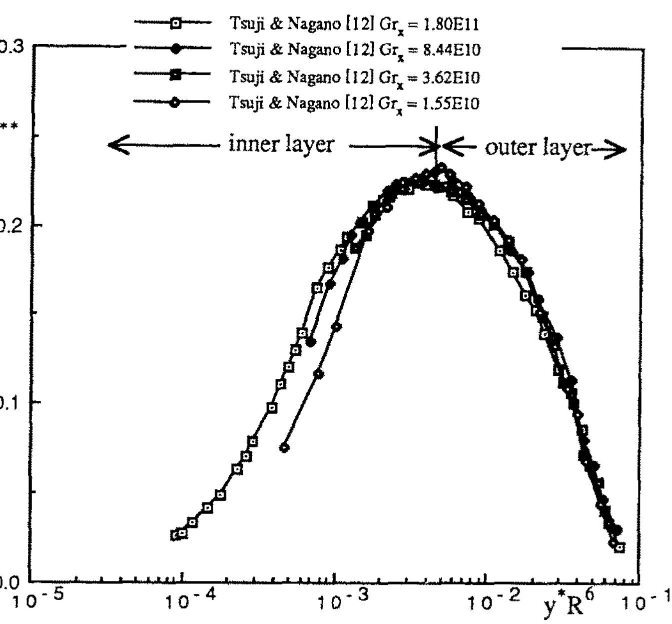


FIG. 4. The correlation of the velocity profile in the turbulent natural convection boundary layer along vertical plates. (a) The correlation of the velocity profile in the inner sublayer. (b) The correlation of the velocity profile in the outer sublayer.

the inner sublayer and when $d = 6$ for the outer sublayer.

By means of curve-fitting, as shown in Fig. 5, we obtain velocity wall function as follows:

$$u^{**} = \min\{f_i, f_o\} \tag{36}$$

$$f_i = 1.41y_i^{**} - 3.11y_i^{**2} + 2.38y_i^{**3} \quad y_i^{**} \leqslant 0.53 \tag{37}$$

$$f_i = 0.228 \quad y_i^{**} > 0.53 \tag{38}$$

$$f_o = 0.228 \quad y_o^{**} < 0.005 \tag{39}$$

(a)

(b)

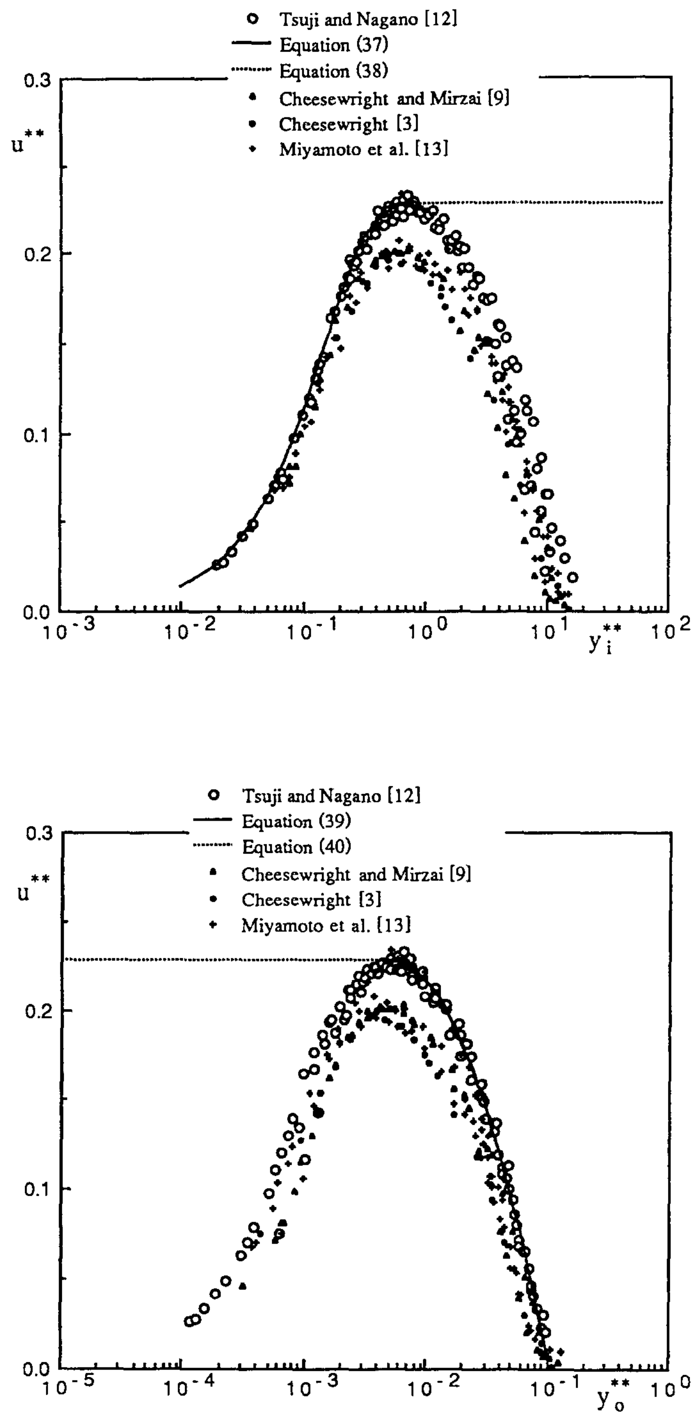


FIG. 5. The velocity wall function of turbulent natural convection along vertical plates. (a) The velocity wall function in the inner sublayer. (b) The velocity wall function in the outer sublayer.

$$f_0 = -0.458-0.258\ln y_0^{**}-0.02425\ln^2 y_0^{**} \quad 0.005 \leqslant y_0^{**} \leqslant 0.1 \quad (40)$$

$$f_0 = 0 \quad y_0^{**} > 0.1 \quad (41)$$

where y_i^{**} and y_0^{**} are the dimensionless distance for inner and outer sublayers, respectively, which are defined as:

$$y_i^{**} \equiv \frac{yu_q^3}{\alpha u_\tau^2} \quad (42)$$

$$y_0^{**} \equiv \frac{\nu u_q^7}{\alpha u_\tau^6}. \quad (43)$$

Let us compare equation (37) with the analytical velocity profile deduced by Plumb and Kennedy [14] and Tsuji and Nagano [12]:

$$u = \frac{\tau_w}{\rho\nu} y - \frac{1}{2}\frac{g\beta(T_w - T_0)}{\nu} y^2 + \frac{1}{3!}\frac{g\beta q_w}{\lambda\nu} y^3 + \frac{1}{\nu}\left[\frac{1}{4!}\left(\frac{\partial^3 \overline{u'v'}}{\partial y^3}\right)_{y=0} y^4 + \cdots\right]. \quad (44)$$

If the higher-order terms can be neglected, equation (44) can be rewritten as:

$$u^{**} = A y_i^{**} + B y_i^{**2} + C y_i^{**3} \quad (45)$$

where:

$$A = Pr^{-1} \quad (46)$$

$$B = -\frac{1}{2}\frac{g\beta\alpha^2(T_w - T_0)}{\nu u_q^3} \quad (47)$$

$$C = \frac{1}{6}\frac{u_\tau^2}{Pr\, u_q^2}. \quad (48)$$

According to equation (27) and the following relationship:

$$\frac{\tau_w}{\rho[g\beta(T_w - T_0)\nu]^{2/3}} = 0.684\, Gr_x^{1/11.9} \quad (49)$$

proposed by Tsuji and Nagano [12], we have:

$$B = \frac{1}{2 \times 0.11^{3/4}\, Pr^{1/2}} \quad (50)$$

$$C = \frac{0.684\, Gr_x^{1/11.9}}{6 \times 0.11^{1/2}}. \quad (51)$$

For air, $A = 1.41$, $B = -3.11$, and $C = 0.3437\, Gr_x^{1/11.9}$. Figure 6 shows the curves of equation (45) with different values of C (corresponding to different magnitudes of Gr_x), from which it can be seen that the last term in the right hand side of equation (45) seems to be negligible in the region of $y_i^{**} < 0.1$. Since both A and B are independent of Gr_x, therefore u^{**} and y_i^{**} are indeed the proper dimensionless parameters to correlate the velocity profile in the near wall region.

For $y_i^{**} > 0.1$, u^{**} is sensitive to the value of C in equation (45). When $Gr_x = 10^{10}$, $C = 2.38$, equation (45) is identical to equation (37). This does not mean that equation (37) is only valid in $Gr_x = 10^{10}$. Equation (37) is a fitted curve based on the experimental data, and hence, it is valid in the same region as the experimental data.

3. DISCUSSION AND CONCLUSION

A comparison of the dimensionless parameters and wall functions between natural and forced convection, Table 1, shows the analogy between natural and forced convection. The boundary layer in forced convection is a constant stress layer where u_τ (known as friction velocity), $T_\tau = q_w/\rho C_p u_\tau$ (known as friction temperature), and ν/u_τ are the proper velocity, temperature, and length scales, respectively, while the inner sublayer in turbulent natural convection is a constant heat flux layer where T_q (called heat flux temperature), $u_Q = (\tau_w/\rho)^2/g\beta\alpha T_q$ (called heat flux velocity), and α/u_q should be the proper temperature, velocity, and length scales, respectively. Therefore y^*, T^*, and u^{**} are the suitable dimensionless parameters to correlate the temperature and velocity profiles in natural convection as y^+, u^+, and T^+ in forced convection.

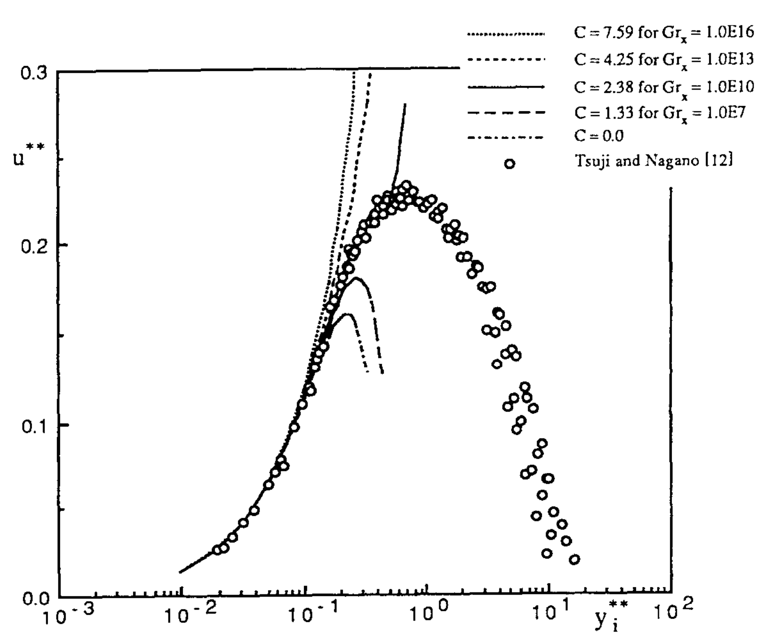


FIG. 6. The curves of equation (45) with $A = 1.41$ and $B = -3.11$.

Table 1. The analogy of the dimensionless parameters between natural and forced convection

Forced convection	Natural convection
$y^+ \equiv \dfrac{yu_\tau}{\nu} = y\left(\dfrac{\tau_w}{\rho\nu^2}\right)^{1/2}$	$y^* \equiv \dfrac{yu_q}{\alpha} = y\left(\dfrac{g\beta q_w}{\alpha^3\rho C_p}\right)^{1/4}$
$u^+ \equiv \dfrac{u}{u_\tau} = u\left(\dfrac{\rho}{\tau_w}\right)^{1/2}$	$T^* \equiv \dfrac{T_w - T}{T_q} = (T_w - T)\left(\dfrac{g\beta\alpha(\rho C_p)^3}{q_w^3}\right)^{1/4}$
$T^+ \equiv \dfrac{(T_w - T)u_\tau}{\left(\dfrac{q_w}{\rho C_p}\right)} = \dfrac{\Delta T}{\left(\dfrac{q_w}{\rho C_p}\right)}\left(\dfrac{\tau_w}{\rho}\right)^{1/2}$	$u^{**} \equiv \dfrac{ug\beta\alpha T_q}{\left(\dfrac{\tau_w}{\rho}\right)^2} = \dfrac{u}{\left(\dfrac{\tau_w}{\rho}\right)^2}\left(\dfrac{g\beta\alpha q_w}{\rho C_p}\right)^{3/4}$

It can be seen from the scales of temperature and velocity that T_q is independent of τ_w, while u_Q is dependent both on τ_w and q_w. So, the correlation of temperature data is independent of the wall shear stress and the correlation of velocity data is dependent both on the wall shear stress and the wall heat flux, which is consistent with the opinion of Cheesewright and Mirzai [9]. It is interesting to compare the correlation in forced convection where velocity data are independent of the wall heat flux, while temperature data are dependent both on the wall shear stress and the wall heat flux.

Because only fluid properties and local parameters are included, without the ambient temperature, the proposed wall functions are expected to be valid in non-isothermal environments or flows in cavities and not only on vertical flat plates. They are suitable for numerical simulation since they do not contain additional parameters such as maximum velocity and boundary thickness which cannot be obtained accurately during the calculation unless a very fine grid system is applied.

Acknowledgements—The authors are grateful to the Swiss Federal Office of Energy (BEW) for providing partial financial support to the investigation. Particular thanks are given to Dr Qingyan Chen and Dr Alois Schaelin for their helpful discussions.

REFERENCES

1. M. Skovgaard and P. V. Nielsen, Numerical investigation of transient flow over a backward step using a low Reynolds number k–ε model, *Air Movement and Ventilation Control within Buildings, Proc. 12th AIVC Conf.*, Ottawa, pp. 202–212 (1991).
2. Q. Chen and Z. Jiang, Significant questions in predicting room air motion, *ASHRAE Trans.* **98,** Paper AN-92-9-1 (1992).
3. R. Cheesewright, Turbulent natural convection from a vertical plane surface, *J. Heat Transfer* **90,** 1–8 (1968).
4. T. Fujii, M. Takeuchi, M. Fujii, K. Suzaki and H. Uehara, Experiments on natural-convection heat transfer from the outer surface of a vertical cylinder to liquids, *Int. J. Heat Mass Transfer* **13,** 753–787 (1970).
5. M. Miyamoto and M. Okayama, An experimental study of turbulent free convection boundary layer in air along a vertical plate using LDV, *Bull. JSME* **25,** 1729–1736 (1982).
6. T. Tsuji and Y. Nagano, Velocity and temperature measurements in a natural convection boundary layer along a vertical flat plate, *Experimental Thermal and Fluid Science* **2,** 208–215 (1989).
7. R. A. W. M. Henkes, Natural-convection boundary layer, Ph.D. Thesis, Delft University (1990).
8. W. K. George and S. P. Capp, A theory for national convection turbulent boundary layer next to heated vertical surfaces, *Int. J. Heat Mass Transfer* **22,** 813–826 (1979).
9. R. Cheesewright and M. H. Mirzai, The correction of experimental velocity and temperature data for a turbulent natural convection boundary layer, *Proc. 2nd U.K. Natn. Conf. on Heat Transfer*, Glasgow, pp. 79–89 (1988).
10. A. Moser, Numerical simulation of room thermal convection—review of IEA Annex-20 results, *Proc. International Symposium on Room Air Convection and Ventilation Effectiveness*, Tokyo, Japan, pp. 76–96, 22–24 July (1992).
11. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*. Academic Press, New York (1959).
12. T. Tsuji and Y. Nagano, Characteristics of a turbulent natural convection boundary layer along a vertical flat plate, *Int. J. Heat Mass Transfer* **31,** 1723–1734 (1988).
13. M. Miyamoto, Y. Katoh, J. Kurima and H. Kajino, An experimental study of turbulent free convection boundary layer along a vertical surface using LDV, *The Application of Laser Doppler Velocimetry, Proceedings of the Osaka Symposium on Flow Measuring-Techniques*, p. 82, 15 July (1983).
14. O. A. Plumb and L. A. Kennedy, Application of a k–ε turbulence model to natural convection from a vertical isothermal surface, *J. Heat Transfer* **99,** 79–85 (1977).